A Phenomenological Expression for Deuteron Electromagnetic Form Factors Based on Perturbative QCD Predictions*

Hui-fang WU[1),2)], Jun CAO[1)] and Tao HUANG[1)]

1) Institute of High Energy Physics, P.O. Box 918(4), Beijing, 100039, China.

and

2) Institute of Theoretical Physics, P.O.Box 2735, Beijing, 100080, China.

Abstract

For deuteron electromagnetic form factors,perturbative QCD(pQCD) predicts that G^+_{00} becomes the dominate helicity amplitude and that G^+_{+0} and G^+_{+-} are suppressed by factors $\Lambda_{\rm QCD}/Q$ and $\Lambda^2_{\rm QCD}/Q^2$ at large Q^2,respectively. We try to discuss the higher order corrections beyond the pQCD asymptotic predictions by interpolating an analytical form to the intermediate energy region. From fitting the data,our results show that the helicity-zero to zero matrix element G^+_{00} dominates the gross structure function $A(Q^2)$ in both of the large and intermediate energy regions; it is a good approximation for G^+_{+-} to ignore the higher order contributions and the higher order corrections to G^+_{+0} should be taken into account due to sizeable contributions in the intermediate energy region.

PACS numbers: 13.40.Gp,24.85.+p,12.38.Bx,27.10.+h

Key words: QCD, deuteron, electromagnetic form factor

*The Project Supported by the National Science Foundation of China (NSFC)and Grant of Academia Sinica.

1 Introduction

It was found a long time ago that the traditional meson-nucleon picture can not explain the form factors of the deuteron as the momentum transfer $Q^2 > 1$ GeV^2 [1]. It means that the fundamental degrees of freedom of QCD, the quark and gluon degrees of freedom, must be taken into account. However, A pure perturbative QCD (PQCD) calculation[2] shows that the theoretical prediction is much smaller than the data at currently accessible energies, although it may be correct in very large Q^2. To explain the deuteron form factors in the intermediate energy region, we have suggested a QCD-inspired model for the helicity-zero to zero matrix elements G^+_{00} in the light-cone frame[3], which should be the dominant amplitude from PQCD predictions[4]. This model can explain the data of the deuteron electromagnetic structure function $A(Q^2)$ and shows that G^+_{00} is already dominant at Q^2 of 1 GeV^2. Furthermore, it was found that G^+_{+0} can not be neglected in the form factor $G_M(Q^2)$[5,6] and G^+_{+-} plays an important role in $G_Q(Q^2)$[6]. Neglecting them will result in contradictions with both the data and the conventional meson-nucleon picture in the low energy region.

Perturbative QCD (PQCD) predicts that G^+_{00} becomes the dominant helicity amplitude at large Q^2 and that G^+_{+0} and G^+_{+-} are suppressed by factors $\Lambda_{\rm QCD}/Q$ and $\Lambda^2_{\rm QCD}/Q^2$, respectively. neglecting G^+_{+0} and G^+_{+-} contributions at large Q^2, we have the relation approximately,

$$G_C : G_M : G_Q = (1 - \frac{2}{3}\eta) : 2 : -1, \tag{1}$$

where $\eta = Q^2/4M^2$ and M is the mass of the deuteron. However, the helicity-flip amplitude G^+_{+0} and G^+_{+-} contribute to G_M and G_Q because of the kinematic enhancement in the intermediate energy region. In order to explore the role of G^+_{+0} and G^+_{+0}, we have tried to expand them to the second order in $\Lambda_{\rm QCD}/M$[6], according to the QCD predictions at large Q^2. This expansion can connects smoothly PQCD predictions in the high energy region with traditional nuclear physics predictions in the low energy region. It was shown that the second order contribution strongly affects the behavior of G_Q in the intermediate energy region. At large Q^2, the ratio of form factors (1) is slightly modified.

Following this approach, G^+_{+0} and G^+_{+-} will be expanded to higher orders (beyond the second order) in $\Lambda_{\rm QCD}/M$ according to the PQCD prediction at large Q^2. In order to explore the role of higher order contributions we discuss the possibility to interpolate an expression for G^{+}_{+0} and G^{+}_{+-} to the intermediate energy region in this paper. It is worthwhile to unify the predictions for the deuteron form factors from the low energy to large energy region.

A general consideration based on the PQCD predictions for the deuteron from factors is given in Sec. 2. As an example, a phenomenological analytic form including the higher order corrections in $\Lambda_{\rm QCD}/M$ is suggested in Sec. 3. The numerical results and summary are presented in Sec. 4 and Sec. 5, respectively.

2 A General Consideration Based on the PQCD Predictions

For the deuteron case, the matrix element of the electromagnetic current J^μ is defined as

$$G^{\mu}_{\lambda'\lambda} = \langle P'\lambda' \mid J^\mu \mid P\lambda\rangle \ , \tag{2}$$

where $Q^2 = -(P'-P)^2$ and $|P\lambda\rangle$ is an eigenstate of the deuteron with momentum P and helicity λ. In the standard light-cone frame (LCF), defined by Ref.[7] $q^+ = 0, q_y = 0$, and $q_x = Q$, the charge, magnetic, and quadrapole form factors can be obtained from the plus component of three helicity matrix elements:

$$G_C = \frac{1}{2p^+(2\eta+1)}\left[(1-\frac{2}{3}\eta)G^{+}_{00} + \frac{8}{3}\sqrt{2\eta}G^{+}_{+0} + \frac{2}{3}(2\eta-1)G^{+}_{+-}\right], \tag{3a}$$

$$G_M = \frac{1}{2p^+(2\eta+1)}\left[2G^{+}_{00} + \frac{2(2\eta-1)}{\sqrt{2\eta}}G^{+}_{+0} - 2G^{+}_{+-}\right] \tag{3b}$$

and

$$G_Q = \frac{1}{2p^+(2\eta+1)}\left[-G^{+}_{00} + \sqrt{\frac{2}{\eta}}G^{+}_{+0} - \frac{\eta+1}{\eta}G^{+}_{+-}\right]. \tag{3c}$$

In terms of G_C, G_M, and G_Q, the Rosenbluth cross section and the tensor polarization T_{20} for elastic ed scattering can be expressed as

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott}\left[A(Q^2) + B(Q^2)\tan^2(\frac{\theta}{2})\right] \ , \tag{4}$$

and

$$T_{20} = -\frac{\frac{8}{9}\eta^2 G_Q^2 + \frac{8}{3}\eta G_C G_Q + \frac{2}{3}\eta G_M^2\left[\frac{1}{2} + (1+\eta)\tan^2(\frac{\theta}{2})\right]}{\sqrt{2}\left[A + B\tan^2(\frac{\theta}{2})\right]}, \tag{5}$$

where $A(Q^2)$ and $B(Q^2)$ are given by

$$A(Q^2) = G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 G_Q^2 \tag{6}$$

and

$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2. \tag{7}$$

It was shown [4] that, in LCF, the helicity-zero to zero matrix element G_{00}^+ would be the dominant helicity amplitude at large Q^2 for elastic *ed* scattering from the PQCD predictions. It means the G_{00}^+ dominance in the structure function $A(Q^2)$. It was also argued[8] that the dominance of G_{00}^+ begins at $Q^2 \gg 2M\Lambda_{\rm QCD} \sim 0.8$ GeV^2 but not $\eta >> 1$. Thus $2M\Lambda_{\rm QCD}$ is a scale of validity of PQCD predictions and the quark and gluon degrees of freedom in the deuteron should be taken into account to solve the problem that the experimental results of $A(Q^2)$ are in sharp disagreement with the meson exchange calculations for $Q^2 > 0.8$ GeV^2[1]. To make detailed prediction for G_{00}^+, we have suggested a QCD-inspired model[3] in the region of $Q^2 > 1$ GeV^2 based on the reduced form factor method[9], which fit the data well.

PQCD predicts that G_{+0}^+ and G_{+-}^+ are suppressed by factors $\Lambda_{\rm QCD}/Q$ and $\Lambda_{\rm QCD}^2/Q^2$, respectively. However, in the intermediate energy region, G_{00}^+ dominates the charge form factor G_C, but not G_M and G_Q. As shown in Eq.(3), while $\eta < \frac{1}{2}$, the G_{+0}^+ contribution to G_M and G_Q are enhanced by a factor $\frac{1}{\sqrt{2\eta}}$ and G_{+-}^+ contribution to G_Q is enhanced by a factor $\frac{1}{2\eta}$. Although G_{+0}^+ and G_{+-}^+ are suppressed for dynamic reason, they contribute significantly to G_M and G_Q because of the kinematic enhancement. Without these contributions, the predicted form factors, except for G_C, are in sharp disagreement with the data. In order to explore the role of G_{+0}^+ and G_{+-}^+, we interpolate a general expression based on perturbative QCD predictions,

$$G_{+0}^+ = \frac{1}{\sqrt{2\eta}} g_{+0}(\eta)\ G_{00}^+ \tag{8a}$$

$$G_{+-}^+ = \frac{1}{2\eta} g_{+-}(\eta)\ G_{00}^+\ , \tag{8b}$$

where $g_{+0}(\eta)$ and $g_{+-}(\eta)$ are any functions of η with $\eta \equiv \frac{Q^2}{4M^2}$. Obviously G_{+0}^+ and G_{+-}^+ are suppressed by factors Λ_{QCD}/Q and Λ_{QCD}^2/Q^2 as long as $g_{+0}(\eta)$ and $g_{+-}(\eta)$ satisfy the following condition,

$$g_{+0}(\eta), \quad g_{+-}(\eta) \rightarrow O(1) \qquad as \qquad \eta \rightarrow \infty \quad . \tag{9}$$

Substituting Eqs.(8) into Eqs.(3), one can get

$$G_c = \frac{1}{2p^+(2\eta+1)}[(1-\frac{2}{3}\eta) + \frac{8}{3}\ g_{+0}(\eta) + \frac{2}{3}(2\eta-1)\frac{1}{2\eta}\ g_{+-}(\eta)]G^+_{00} \quad , \tag{10a}$$

$$G_M = \frac{1}{2p^+(2\eta+1)}[2 + \frac{2\eta-1}{\eta}\ g_{+0}(\eta) - \frac{1}{\eta}\ g_{+-}(\eta)]G^+_{00} \tag{10b}$$

and

$$G_Q = \frac{1}{2p^+(2\eta+1)}[-1 + \frac{1}{\eta}\ g_{+0}(\eta) - \frac{\eta+1}{2\eta^2}\ g_{+-}(\eta)]G^+_{00} \quad . \tag{10c}$$

Now we discuss the constriants on $g_{+0}(\eta)$ and $g_{+-}(\eta)$ from the experimental data. As we know, $G_M(Q^2)$ changes sign at $Q^2 = Q_0^2 \sim 2GeV^2$ [10] (or $\eta = \eta_0 = \frac{Q_0^2}{4M^2} \simeq 0.13$). The dominance of G^+_{00} can not explain this point (see Eq.(3b)) and at least the second term in Eq.(3b) should be in the same order of the first term to cancel it in order to fit data of G_M. That means $g_{+0}(\eta)$ is nonzero. If we first keep $g_{+-}(\eta) = 0$ in Eqs.(10), then $g_{+0}(\eta_0)$ can be determined by the zero at $G_M(\eta_0)$, which turns out to be $g_{+0}(\eta_0) = \frac{2\eta_0}{1-2\eta_0}$. In this case, G_Q is negative at $\eta = \eta_0$ where PQCD begins to be valid. Thus there must be a node in the region $Q^2 < 1GeV^2$ since G_Q is positive at the origin experimentally. The theoretical prediction is contrary to the experimental data without G^+_{+-} contribution. Therefore the predicted form factors are in sharp disagreement with the data without G^+_{+0} and G^+_{+-} contributions and the existence of non-zero $g_{+0}(\eta)$ and $g_{+-}(\eta)$ is necessary. In addition to the constrant (9), $g_{+0}(\eta)$ and $g_{+-}(\eta)$ should astisfy

$$g_{+0}(\eta_0) = \frac{2\eta_0 - g_{+-}(\eta_0)}{1-2\eta_0} \tag{11}$$

to ensure $G_M(\eta_0) = 0$ and

$$2\eta\ g_{+0}(\eta) - 2\eta^2 < (\eta+1)\ \ g_{+-}(\eta) \tag{12}$$

to keep G_Q positive at any momentum transfer. In particular, as $\eta = \eta_0$ Eqs.(11) and (12) give the constraints on $g_{+0}(\eta_0)$ and $g_{+-}(\eta_0)$,

$$g_{+-}(\eta_0) > \frac{2\eta_0^2}{1-\eta_0} \tag{13}$$

and

$$g_{+0}(\eta_0) < \frac{2\eta_0}{1-\eta_0} \tag{14}$$

Eqs.(9), (11) and (12) are three constraints on the functions $g_{+0}(\eta)$ and $g_{+-}(\eta)$.

3 A Phenomenological Example

As mentioned in the section 2, $g_{+-}(\eta) \neq 0$ is important to keep $G_Q > 0$ although $G^+_{+-} = \frac{1}{2\eta} g_{+-}(\eta) G^+_{00}$ is suppressed by the higher order factor $\Lambda^2_{QCD}/Q^2 (= \frac{\Lambda^2_{QCD}}{2M^2} \cdot \frac{1}{2\eta})$. However the G^+_{+0} is the first order correction which makes the zero in G_M at $Q^2_0 \simeq 1.85 GeV^2$. We have expanded G^+_{+0} and G^+_{+-} to the second order in Λ_{QCD}/M in Ref.[6] and numerical results show that the second order contribution to G^+_{+0} plays an important role in the intermediate energy region. In this paper we introduce an exponential form phenomenologically as an example,

$$g_{+0}(\eta) = f \;\; exp(-\frac{bf}{\sqrt{2\eta}}) \tag{15a}$$

and

$$g_{+-}(\eta) = f^2 exp(-\frac{cf}{\sqrt{2\eta}}) \tag{15b}$$

to interpolate the higher order corrections. Obviously, the exponential form (15) satisfies Eq.(9) and is consistent with the perturbative QCD prediction at the large transfer momentum region. Thus Eq.(15) is enable us to make an analytical evaluation to the deuteron form factors.

4 Numerical Results

We input the parameters b and c, and determine f by the zero in G_M. For a certain c, the obtained G_Q increases proportionally with b. We can fix b to connect our predictions with the data smoothly. To retain good convergence, we constrain $b\frac{f}{\sqrt{2\eta}}$ and $c\frac{f}{\sqrt{2\eta}}$ being smaller than unity. On the other hand, we restrict b and c to be positive to keep the exponential damping as Q^2 goes to infinity and it is a resonable assumption, after taking into account the higher order corrections. For $c = 0.0, 0.5$ and -0.5, the predicted $B(Q^2)$, $G_Q(Q^2)$ and $T_{20}(Q^2)$ are shown in figs. (1-3). Experimental data are taken from Refs.[1,10-12]. To smoothly connect with the data of G_Q, the parameter b should be 1.1, 1.3, and 0.8, $f = 0.37, 0.51$, and 0.30, respectively. As argued above, $c = -0.5$ should be abandaned. While $c = 0.5$, the corresponding f is 0.51, which is too large to keep $b\frac{f}{\sqrt{2\eta}}$ smaller than unity as $Q^2 \geq 1$. The parameter $c = 0.0, b = 1.1, f = 0.37$ is an appropriate choice.

5 Summary

Based on perturbative QCD predictions at large momentum transfers we have tried to discuss the corrections to deuteron form factors beyond the second order in Λ_{QCD}/M.A general consideration is given by introducing functions $g_{+0}(\eta)$ and $g_{+-}(\eta)$ and the data at the present energy region put three constrants on the functions $g_{+0}(\eta)$ and $g_{+-}(\eta)$. In order to explore the role of higher order contributions we suggest an exponential form for $g_{+0}(\eta)$ and $g_{+-}(\eta)$ as an example. We conclude that (1) the helicity-zero to zero matrix element G^+_{00} dominates the gross structure function $A(Q^2)$ in both of the large and intermediate energy regions. A QCD-inspired model can describe this matrix element well[3]. (2) G^+_{+0} and G^+_{+-} contributions are important in determining form factors G_M and G_Q; (3) By fitting the data, we get a set of parameters in $g_{+0}(\eta)$ and $g_{+-}(\eta)$, $c = 0.0, b = 1.1, f = 0.37$, which can describe G_M, G_Q and T_{20} appropriately. the parameters $c = 0.0$ indicates that it is good for G^+_{+-} in the intermediate energy region to take the asymptotic behavior which were predicted by pQCD, and the higher order (beyond the second order) contributions are negligible; (4) The higher order corrections to G^+_{+0} should be taken into account and they make sizeable contributions in the intermediate energy region.

References

[1] R.G. Arnold, B.T. Chertok, E.B. Dally, A. Grigorian, C.L. Jordan, W.P. Schütz, R. Zdarko, F. Martin, and B.A. Mecking, Phys. Rev. Lett. **35**, 776 (1975).

[2] G.R. Farrar, K. Huleihel, and H. Zhang, Phys. Rev. Lett. **74**, 650 (1995).

[3] J. Cao and H. Wu, Phys. Rev. C **54**, 1006 (1996).

[4] C.E. Carlson and F. Gross, Phys. Rev. Lett. **53**, 127 (1984).

[5] A.P. Kobushkin and A.I. Syamtomov, Phys. Rev. D **49**, 1637 (1994), Phys. At. Nucl. **58**, 1477 (1995).

[6] J. Cao and H. Wu, Phys. Rev. C **55**, 2191 (1997).

[7] S.D. Drell and T.M. Yan, Phys. Rev. Lett. **24**, 181 (1970).

[8] S.J. Brodsky and J.R. Hiller, Phys. Rev. D **46**, 2141 (1992).

[9] S.J. Brodsky and J.R. Hiller, Phys. Rev. C **28**, 475 (1983).

[10] S. Auffret *et al.*, Phys. Rev. Lett. **54**, 649 (1985);
R.G. Arnold *et al.*, Phys. Rev. Lett. **58**, 1723 (1987);
P.E. Bosted *et al.*, Phys. Rev. C **42**, 38 (1990).

[11] M. Garcon *et al.*, Phys. Rev. C **49**, 2516 (1994).

[12] S. Platchkov *et al.*, Nucl. Phys. **A 510**, 740 (1990).

Figure Captions

Fig.1. Structure function $B(Q^2)$. The dashed dotted line corresponds to the Paris potential calculation. Experimental data are taken from Ref.[10].

Fig.2. The form factor G_Q. Experimental data are taken from Ref.[11].

Fig.3 The tensor polarization T_{20} with scattering angle $\theta = 70°$. The dashed dotted line corresponds to the calculation with Paris potential. Experimental data are taken from Ref.[11]